Math 2E Quiz 4 Afternoon - April 21st

Please write your name and ID on the front.

Show all of your work, and simplify all your answers. *There is a question on the back side.

1. Let B be the ball with radius 5 centered at (0,0,0). Compute

$$\iiint_B (x^2+y^2+z^2)^2 dV.$$

You can leave the exponentiated term as is - you don't need to multiply that term out.

- Since it's the whole ball, $0 \le \rho \le 5$, $\theta \in [0, 2\pi]$, $\phi \in [0, \pi]$,
- $x^2+y^2+z^2 = \rho^2$ too.

+3 bounds +1 +1

Integral Becomes: $\int_{\phi=0}^{\pi} \int_{\theta=0}^{2\pi} \int_{\rho=0}^{5} (\rho^2)^2 \cdot \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$

$$= \int_{\phi=0}^{\pi} \int_{\theta=0}^{2\pi} \int_{\rho=0}^{5} \rho^6 \sin\phi \, d\rho \, d\theta \, d\phi = \int_{\phi=0}^{\pi} \int_{\theta=0}^{2\pi} \frac{\rho^7}{7}\Big|_0^5 \sin\phi \, d\theta \, d\phi$$

θ-indep.

$$= \frac{5^7}{7} \int_{\phi=0}^{\pi} 2\pi \sin\phi \, d\phi = \frac{2\pi}{7} \cdot 5^7 \cdot (-\cos\phi)\Big|_0^{\pi}$$

$$= \boxed{\frac{4\pi}{7} \cdot 5^7}$$

+5

2. Consider R_{xy} to be the trapezoidal region in $\mathbb{R}^2$ with vertices (1,0), (2,0), (0,2), (0,1). Consider

$$\iint_{R_{xy}} \cos\left(\frac{y-x}{y+x}\right) dA.$$

With the change of variable $u = y - x, v = y + x$, we will rewrite this integral in uv-variables.
(i) What is the Jacobian with this transformation?
(ii) Draw the new region S_{uv} from applying this transformation to R_{xy}. Label the boundary curves.
(iii) Using (i) and (ii), write the integral in the uv variables, with correct bounds.
(iv) [*2pts Bonus*] Compute this integral.
You don't need to do part (iv) for full credit on the problem.

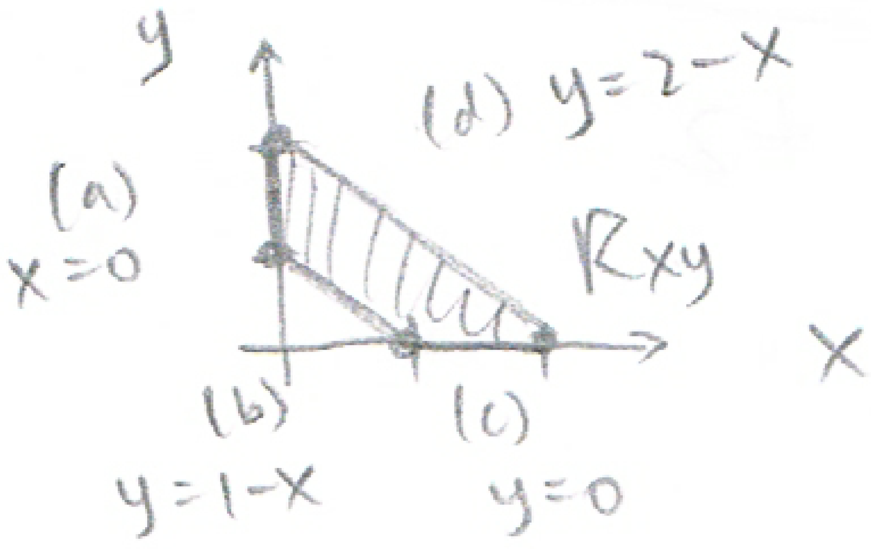
(i) +3 1^{st} need to solve for $x, y \Rightarrow u+v = 2y$, so $y = \dfrac{u+v}{2}$ +1pt

1pt $u - v = -2x$, so $x = \dfrac{v-u}{2}$

Then, $J = \left| \begin{matrix} x_u & x_v \\ y_u & y_v \end{matrix} \right| = \left| \det \begin{bmatrix} -1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix} \right| = \left| -\frac{1}{4} - \frac{1}{4} \right| = \frac{1}{2}$ 1pt

+4

(ii) On xy plane,



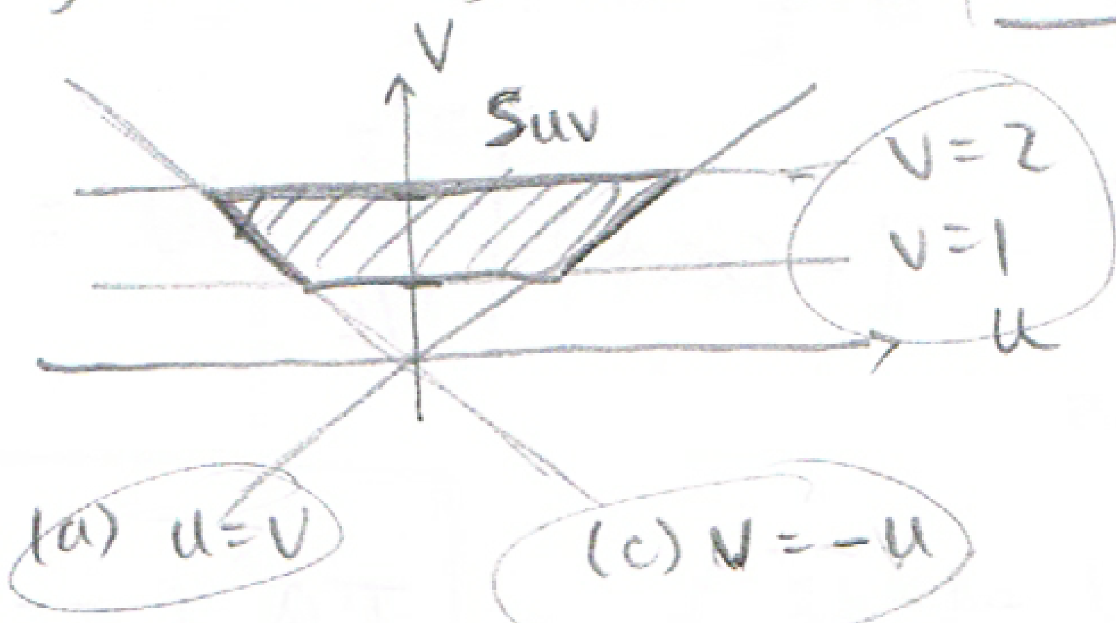
(a) $\Rightarrow$ $x = 0$, $1 \le y \le 2$ and so $u = y$ and $v = y \Rightarrow \boxed{u = v}$, from 1 to 2.

(b) $\Rightarrow$ $y = 1 - x$, $x + y = 1$; so $v = x + y = 1$; $\boxed{v = 1}$.

(c) $\Rightarrow$ $y = 0$ and $1 \le x \le 2$, so $u = -x$, $v = x$ $\Rightarrow \boxed{u = -v}$, or $v = -u$.

(d) $\Rightarrow$ $y = 2 - x$, $x + y = 2$, so $\boxed{v = 2}$ now.

Hence, on uv - plane:



is S_{uv}.

4 pts, 1 for each boundary

+3

(iii) We should integrate left to right, horizontally, and applying $u = y - x$, $v = y + x$,

2pts bounds 1pt

$$\iint_{S_{uv}} \cos\left(\frac{u}{v}\right) J\, dA_{uv} = \int_{v=1}^{v=2} \int_{u=-v}^{u=v} \cos\left(\frac{u}{v}\right) \cdot \frac{1}{2}\, du\, dv$$

(iv) (+2EC) compute,

$$= \int_{v=1}^{2} \frac{1}{2} v \cdot \sin\left(\frac{u}{v}\right)\Big|_{u=-v}^{u=v} dv = \frac{1}{2} \int_{v=1}^{v=2} v \cdot \underbrace{\left(\sin(1) - \sin(-1)\right)}_{\text{is } 2 \cdot \sin(1)}$$

because sine ~ odd,

$$= \frac{2\sin(1)}{2} \int_1^2 v\, dv = \frac{\cancel{2}\sin(1)}{2} \cdot \frac{v^2}{\cancel{2}} \Big|_1^2 = \boxed{\frac{3\sin(1)}{2}}$$

(2pts EC)